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Width of exotics from QCD sum rules : tetraquarks or molecules?

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We investigate the widths of the recently observed charmonium like resonances $X(3872)$, $Z(4430)$ and $Z_2(4250)$ using QCD sum rules. Extending previous analyses regarding these states as diquark-antiquark states or molecules of D mesons, we introduce the Breit-Wigner function in the pole term. We find that introducing the width increases the mass at small Borel window region. Using the operator-product expansion up to dimension 8, we find that the sum rules based on interpolating current with molecular components give a stable Borel curve from which both the masses and widths of these resonances can be well obtained. Thus the QCD sum rule approach strongly favors the molecular description of these states.

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I. INTRODUCTION

Since the discovery of a charmonium like resonance $X(3872)$ by Belle Collaboration [1], plenty of similar resonances have been observed in the decay of B mesons. In particular, the recently observed $Z^+(4430)$ in $\pi^+\psi'$ invariant mass spectrum has a charge [2], and consequently can not be a simple charmonium. There are already a number of interpretations on the structure of these resonances [3]. Although the nature of these states is still an open question, tetraquark state and molecular state are both intriguing possibilities. Since the decay products are a charmonium and a pion, it is natural to expect that the parent contains four quarks including c and $\bar{c}$. On the other hand, the masses of $X(3872)$, $Z^+(4430)$ and the recently observed resonancelike structure $Z_2^+(4250)$ [4], are very close to thresholds of two D-meson states, D^*D, D^*D_1 and D_1D, respectively, and, therefore, it is very tempting to interpret these states as molecular states.

Motivated by these facts, QCD sum rules (QCDSR) have been extensively used to study these resonances. In Ref. [5], $X(3872)$ was analyzed by assuming it to be a $J^{PC}=1^{++}$ tetraquark ($c\bar{c}q\bar{q}$) state. Although the result agreed with the experimental data, an analysis using a current composed of a D^*D molecule shows better operator-product expansion (OPE) convergence and closer agreement with experimentally observed mass [6], strongly suggesting a molecular nature of $X(3872)$.

In Ref. [7], $Z^+(4430)$ was considered as a D^*D_1 molecule and a good agreement with data was obtained, while the tetraquark description has been found to be unsatisfactory [8]. Similarly, we have applied the molecular description to the most recent data of $Z_1^+(4050)$ and $Z_2^+(4250)$ [4], and found that a D_1D molecular state can be attributed to $Z_2^+(4250)$. However, it was not possible to explain $Z_1^+(4050)$ as a molecular D^*D^* state [9]. In all of the calculations above, however, small but finite width was not taken into account, in spite of that fact that $Z^+(4430)$ and $Z_2^+(4250)$ have widths $\Gamma_{Z^+(4430)} = 45^{+18+30}_{-13-13}$ MeV [2] and $\Gamma_{Z_2^+(4250)} = 177^{+54+316}_{-39-61}$ MeV [4], respectively. Of course these widths are much smaller than their masses, around 4 GeV. However, the effect of the width should be examined in order to clarify the structure of these states.

In this paper, we extend the previous analyses [5, 6, 7, 8, 9] to include the effect of finite width and give further consideration on the possibility that these states can be considered as tetraquarks or molecules. The width is usually not calculated in QCD sum rule approaches as the OPE are usually restricted to three terms; perturbative, dimension four and dimension six terms. Hence the phenomenological sides are composed of three unknown parameters; mass, continuum, and overlap constant. In the present analysis, the OPE are composed of operators with four different dimensions. Therefore, an analysis including the width is sensible.

In the next section, we give a brief review of our QCDSR analyses. In Sec. III, we discuss some general features of effect of finite width. Quantitative analyses of the exotic states are given in Sec. IV. Section V is devoted to the summary.

II. QCD SUM RULES

The QCD sum rules for mesons are based on the two-point function of a current $j(x)$ describing a desired state

$$\Pi(q) = i\int d^4x e^{iq\cdot x}\langle 0|T[j(x)j^\dagger(0)]|0\rangle, \qquad (1)$$

and the dispersion relation

$$\Pi(q) = \int ds \frac{\rho(s)}{s-q^2} + (\text{subtraction terms}) \qquad (2)$$

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with $\rho(s)$ being the spectral density. While computing the two-point function in terms of quarks and gluons by making use of operator product expansion (OPE), which takes into account non-perturbative effect through QCD condensates, one models the hadronic spectral density $\rho^{\rm phen}(s)$ with a pole describing the ground state and a continuum, namely,

$$\rho^{\rm phen}(s) = \rho^{\rm pole}(s) + \rho^{\rm cont}(s). \tag{3}$$

In the narrow width approximation, the pole part of the hadronic spectral density is set to a delta function $\rho^{\rm pole}(s) = \lambda^2\delta(s - m^2)$, with $\langle 0|j|{\rm meson}\rangle = \lambda$ being the overlap of the current and the physical meson. In this work, we replace this part with the relativistic Breit-Wigner function to take the width into account. The continuum part above the threshold s_0 is given by the result obtained with the OPE

$$\rho^{\rm cont}(s) = \rho^{\rm OPE}(s)\theta(s - s_0), \tag{4}$$

with $\theta(x)$ being the step function. The OPE side is calculated up to leading order in α_s and condensates up to dimension eight are considered. Currents and OPE terms used in this work are taken from Ref. [5, 6, 7, 8, 9]. The correlation function in the OPE side can be expressed as

$$\Pi^{\rm OPE}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho^{\rm OPE}(s)}{s - q^2} + \Pi^{{\rm mix}\langle\bar{q}q\rangle}(q^2), \tag{5}$$

where $\rho^{\rm OPE}(s) = \pi^{-1}{\rm Im}\Pi^{\rm OPE}(s)$. Then, we can extract the pole term by equating the OPE expression and the phenomenological expression, making the Borel transformation on both sides and then transferring the continuum contribution (4) to the OPE side. The sum rule is hence given by

$$\int_{4m_c^2}^{\infty} ds\, e^{-s/M^2}\rho^{\rm pole}(s) = \int_{4m_c^2}^{s_0} ds\, e^{-s/M^2}\rho^{\rm OPE}(s) + \Pi^{{\rm mix}\langle\bar{q}q\rangle}(M^2). \tag{6}$$

Note that the left-hand side becomes $\lambda^2 e^{-m^2/M^2}$ in the narrow width approximation. In this case, one can extract the pole mass by taking the ratio between the derivative of Eq. (6) with respect to $1/M^2$ and Eq. (6) itself. In the present work, we introduce the width by employing the Breit-Wigner function to the pole contribution

$$\rho^{\rm pole}(s) = \frac{1}{\pi}\frac{f\Gamma\sqrt{s}}{(s - m^2)^2 + s\Gamma^2}. \tag{7}$$

The mass and width are determined by looking at the stability of mass against varying Borel mass M^2, as usual. The relevant Borel window is determined by the convergence of the OPE for the minimum $M^2_{\rm min}$ and the pole dominance criterion for the maximum $M^2_{\rm max}$. As usual, we determine $M^2_{\rm min}$ by requiring the dimension eight condensate contribution to be less than 10% of the total OPE and $M^2_{\rm max}$ from more than 50% pole dominance. We calculate the mass by fixing a width and solving the equation for the ratio

$$-\frac{1}{\Pi^{\rm OPE}(M^2)}\frac{\partial\Pi^{\rm OPE}(M^2)}{\partial(1/M^2)} = \frac{\int_{4m_c^2}^{\infty} dsse^{-s/M^2}\rho^{\rm pole}(s)}{\int_{4m_c^2}^{\infty} dse^{-s/M^2}\rho^{\rm pole}(s)} \tag{8}$$

where $\Pi^{\rm OPE}(M^2)$ is the right-hand side of Eq. (6) and $\rho^{\rm pole}(s)$ is given by Eq. (7).

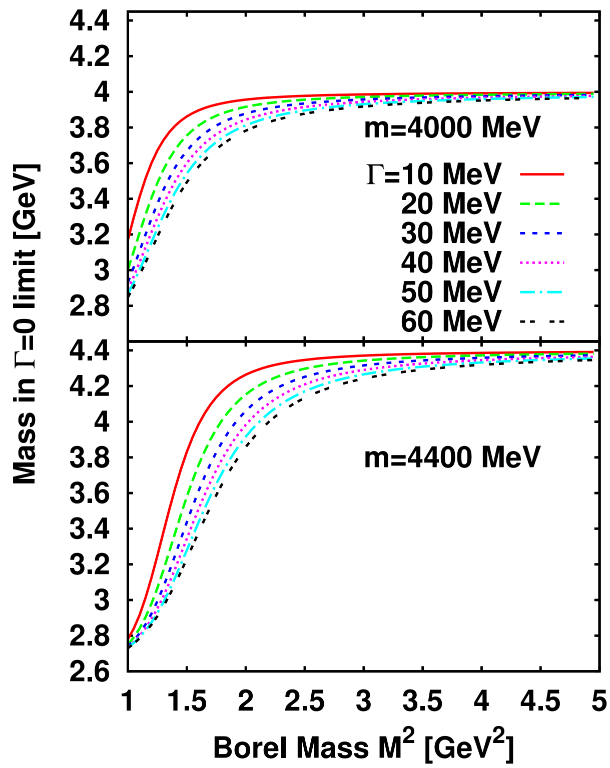

FIG. 1: (color online). Right-hand side of Eq. (8) as a function of Borel mass M^2 with fixed m and Γ. Upper and lower panels stand for the case of $m = 4$ GeV and $m = 4.4$ GeV, respectively.

III. GENERAL FEATURES

Since the left-hand side in Eq. (8) does not change by introducing a width, one can investigate how the mass changes only by looking at the behavior of the right-hand side.

Figure 1 shows the right-hand side of Eq. (8) as a function of Borel mass M^2. One can see that it is a monotonic function of M^2 if both m and Γ are fixed. It rapidly increases at small M^2 and then asymptotically reaches to the Breit-Wigner mass m. In analyses of QCDSR, we solve Eq. (8) with a value of left-hand side of Eq. (8) given by the OPE side and the continuum. This procedure corresponds to finding an intersection between a

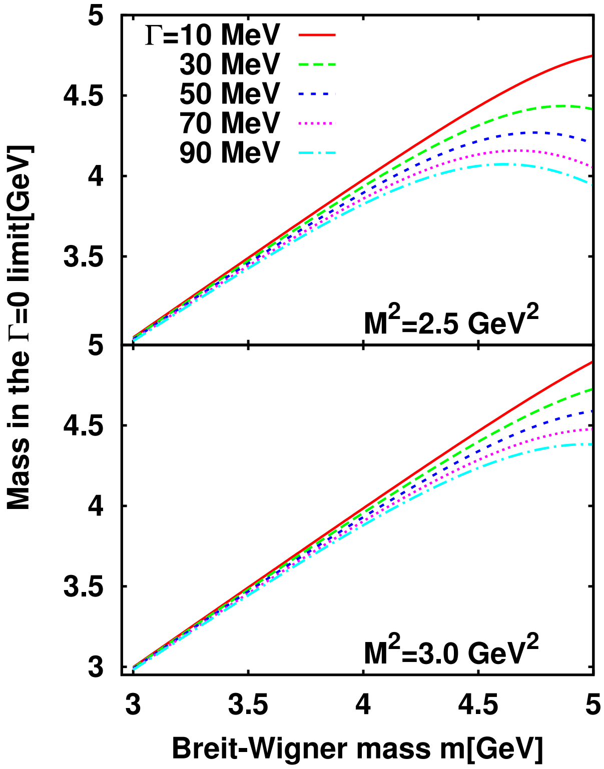

FIG. 2: (color online.) Same as Fig. 1, but as a function of the Breit-Wigner mass and for a fixed Borel mass.

horizontal line denoting the value of mass in the $\Gamma = 0$ and the curves of a given Borel mass in the figures. For example, if one has $m = 3.8$ GeV in the $\Gamma = 0$ case, a possible solution at $M^2 = 2.0$ GeV^2 and $m = 4$ GeV is $\Gamma \simeq 40$ MeV. If one sets $m = 4.4$ GeV, $\Gamma \simeq 60$ MeV is one of the possible solutions. The best solution is determined by looking at the stability against M^2. From the monotonic behavior seen in Fig. 1, one notes that introducing the width increases the mass especially at small Borel mass region. Hence, if one gets the mass in the $\Gamma = 0$ case which monotonically increases as M^2 increases, it will be improved by including the width. This fact gives a guideline on the QCDSR analyses.[1]

We also plot the direct relation between the Breit-Wigner mass and the mass in the $\Gamma = 0$ case in Fig. 2. Here we fixed the Borel mass $M^2 = 2.5$ GeV^2 in the top panel and 3.0 GeV^2 in the bottom panel, which are typical values satisfying the stability criterion in the QCDSR analyses below. One can see that deviation from the mass in the $\Gamma = 0$ is larger for larger mass m and smaller Borel mass M^2. One should note that it is no longer monotonic as a function of m at smaller M^2 and large Γ, as seen in the top panel. This means that if one gets the mass 4200 MeV in the $\Gamma = 0$ case with stability at $M^2 = 2.5$ GeV^2, the maximum width of this state is limited to 50 MeV. This also gives the constraint on possible mass and width values.

[1] This is not a completely general result of QCDSR; as seen in [10], introducing width leads to smaller mass when M^2 is large compared to m.

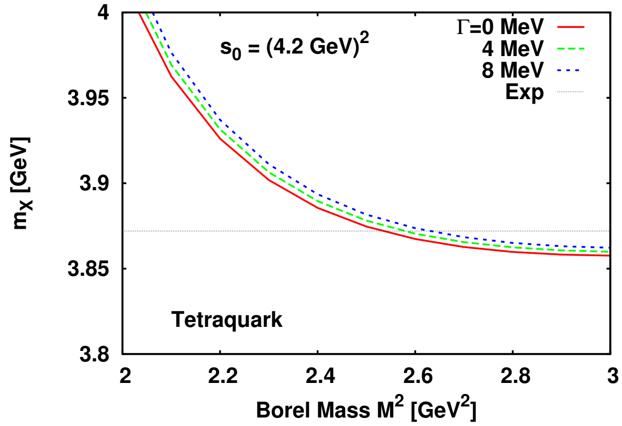

FIG. 3: (color online.) Results for $X(3872)$ as a tetraquark state with width. Continuum thresholds s_0 is taken to be $\sqrt{s_0} = 4.2$ GeV.

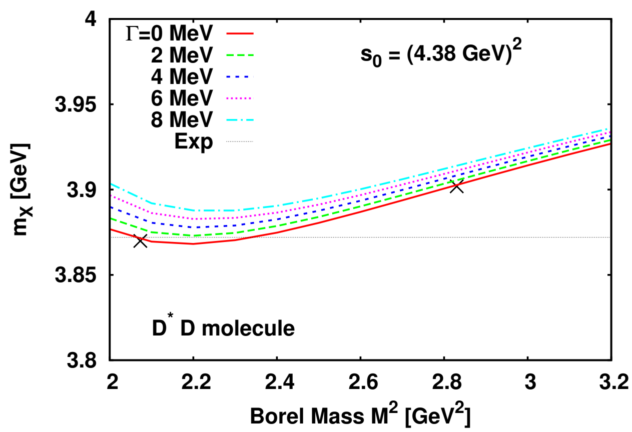

FIG. 4: (color online). Results for $X(3872)$ as a D^*D molecule. The crosses indicate lower and upper limit of the Borel window, respectively.

IV. RESULTS

For parameters in the QCDSR analyses, we use the same parameter set as in the previous works and assume the factorization of the higher dimensional condensates. Namely, $m_c = 1.23$ GeV, $\langle \bar{q}q \rangle = -(0.23\text{GeV})^3$, $\langle g^2G^2 \rangle = 0.88\text{GeV}^4$, $\langle \bar{q}g\sigma \cdot Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ and $m_0^2 = 0.8\text{GeV}^2$. Here we ignored the possible uncertainties in these parameters. Because the uncertainties are those related to the OPE side, which are unchanged by including the width,

possible errors on masses will not be so different from the ones estimated in the previous works.

From the consideration in the previous section, one finds that tetraquark configurations for $Z(4430)$ examined in Ref. [8] are ruled out even if the width is taken into account. In the $J^P = 0^-$ results (Fig. 3 of Ref. [8]), the mass in the $\Gamma = 0$ case shows a good stability. Incorporating width does not improve the stability, but it raises the value of mass, which is already bigger than the experimental value in the $\Gamma = 0$ case. If one assumes $J^P = 1^-$ tetraquark state, it is shown that the mass is more than 300 MeV larger than the experimental value, and the functional behavior with respect to M^2 is monotonically decreasing [8]. Both of these features are only worsened by introducing the width in the calculations. The failure to explain the $Z(4430)$ with tetraquark structures cannot be corrected by taking width into account. In the case of $X(3872)$, the value of mass in the $\Gamma = 0$ limit is found to be in agreement with the data [5] while functional behavior against M^2 is monotonically decreasing. In this case, however, the experimental width has been found to be around 2.3 MeV [1]. Then, such a small width hardly improves the Borel curve, as shown in Fig. 3.[2]

Contrary to the sum rules obtained with interpolating currents based on large tetraquark component, molecular descriptions are much more promising. First, let us consider a counter example of Fig. 3. The current for D^*D molecule is obtained by replacing the strange quark by a light quark in Ref. [6]:

$$j_\mu = \frac{i}{\sqrt{2}} \left[(\bar{u}_a \gamma_5 c_a)(\bar{c}_b \gamma_\mu u_b) - (\bar{c}_a \gamma_5 u_a)(\bar{u}_b \gamma_\mu c_b) \right] \ , \quad (9)$$

where a and b are color indices. The combination $D^0 \bar{D}^{*0} - \bar{D}^0 D^{*0}$ has $J^{PC} = 1^{++}$ as the $X(3872)$ meson. To show that this current has positive charge conjugation we notice that under charge conjugation transformation the two terms in Eq. (9) transforms as:

$$\begin{aligned} \hat{C}[(\bar{u}_a \gamma_5 c_a)(\bar{c}_b \gamma_\mu u_b)]\hat{C}^{-1} &= -[(\bar{c}_a \gamma_5 u_a)(\bar{u}_b \gamma_\mu c_b)] \\ \hat{C}[(\bar{c}_a \gamma_5 u_a)(\bar{u}_b \gamma_\mu c_b)]\hat{C}^{-1} &= -[(\bar{u}_a \gamma_5 c_a)(\bar{c}_b \gamma_\mu u_b)] \end{aligned} \quad (10)$$

Therefore, one obtains

$$\hat{C} j_\mu \hat{C}^{-1} = j_\mu . \quad (11)$$

The symmetrical combination, $D^0 \bar{D}^{*0} + \bar{D}^0 D^{*0}$, would provide exactly the same mass, within our sum rule approach.

The resultant OPE expressions for D^*D molecule are obtained by putting $m_s = 0$ and $\langle \bar{s}s \rangle = \langle \bar{q}q \rangle$ in the expressions in Ref. [6]. Figure 4 shows the result of mass of the D^*D molecular state with $\Gamma = 0, 2, 4, 6, 8$ MeV. One sees that the molecular description gives better stability than the tetraquark case. Furthermore, although effect of width is not larger, we can fit the experimental mass 3872 MeV and width $\Gamma < 2.3$ MeV simultaneously with a continuum threshold $\sqrt{s_0} = 4.38$ GeV, and obtain a broad Borel window. Consequently, it is strongly favored that $X(3872)$ is a D^*D molecular state.

Next, we consider $Z(4430)$ states as a D^*D_1 molecule. The current and OPE expressions are given in Ref. [7]. It has been shown that the molecular description gives a mass which agrees with the experiment well. Since the mass in the $\Gamma = 0$ case is monotonically increasing function of M^2, it is expected that incorporating the width improves the stability according to the result shown in Sec. III.

Figure 5 shows the result of D^*D_1 molecule with various continuum thresholds. In Ref. [7], the continuum threshold is determined as $\sqrt{s_0} = 4.8 - 5.0$ GeV. The center value $\sqrt{s_0} = 4.9$ GeV case is plotted in the right-bottom panel. In this case, the mass in the $\Gamma = 0$ case agrees well with the experimental value. As introducing the width raises the mass, however, the mass becomes larger than experiment when Γ is as large as the experiment. One notes that the stability becomes much better when $\Gamma \simeq 30$ MeV. Other three panels show the cases with lower continuum thresholds. Especially $\sqrt{s_0} = 4.6$ GeV case reproduces both mass and width quite well. One can see that $\Gamma \simeq 40$ MeV gives the best stability of the mass, which perfectly agrees with the the experiment. In the lower continuum thresholds case, however, we have to relax the criterion for the allowed region of sum rule analyses, i.e., Borel window. Since the mass of $Z(4430)$ is close to D^*D_1 threshold, it might be plausible that the continuum contribution becomes larger when D^*D_1 forms a molecule, as assumed in this calculation. The arrows in the figure indicate the Borel masses determined from various values of the relative contributions of the dimension eight condensates (for $M^2_{\rm min}$, upward arrows) and the continuum contribution (for $M^2_{\rm max}$, downwards arrows). One sees that reasonable Borel windows open if one relaxes the condition for either $M^2_{\rm min}$ or $M^2_{\rm max}$, or both of them.

One notes that there is a truncation of the curves in each panel, especially for large width data. This is due to the nature of the Breit-Wigner function, shown in upper panel of Fig. 2, that the right-hand side of Eq. (8) has the maximum at low Borel mass and large width region. This fact appears as an absence of the solution of Eq. 8 for a fixed Borel mass and Γ. Hence, it expresses a maximum width allowed by the QCDSR for each value of the continuum threshold.

Finally we consider the recently discovered $Z_2^+(4250)$ as a D_1D molecule. The current and resultant OPE expressions are given in Ref. [9]. In Ref. [9], it is shown that D_1D molecular description gives a reasonble agreement with the $Z_2^+(4250)$ mass. As mentioned above, this state has a large width whose effect should be examined.

[2] In Ref. [5], the mass of $X(3872)$ was evaluated by including condensates up to dimension five and higher dimensional ones were used to estimate errors. In this paper, we have included all the condensates given in Ref.[5].

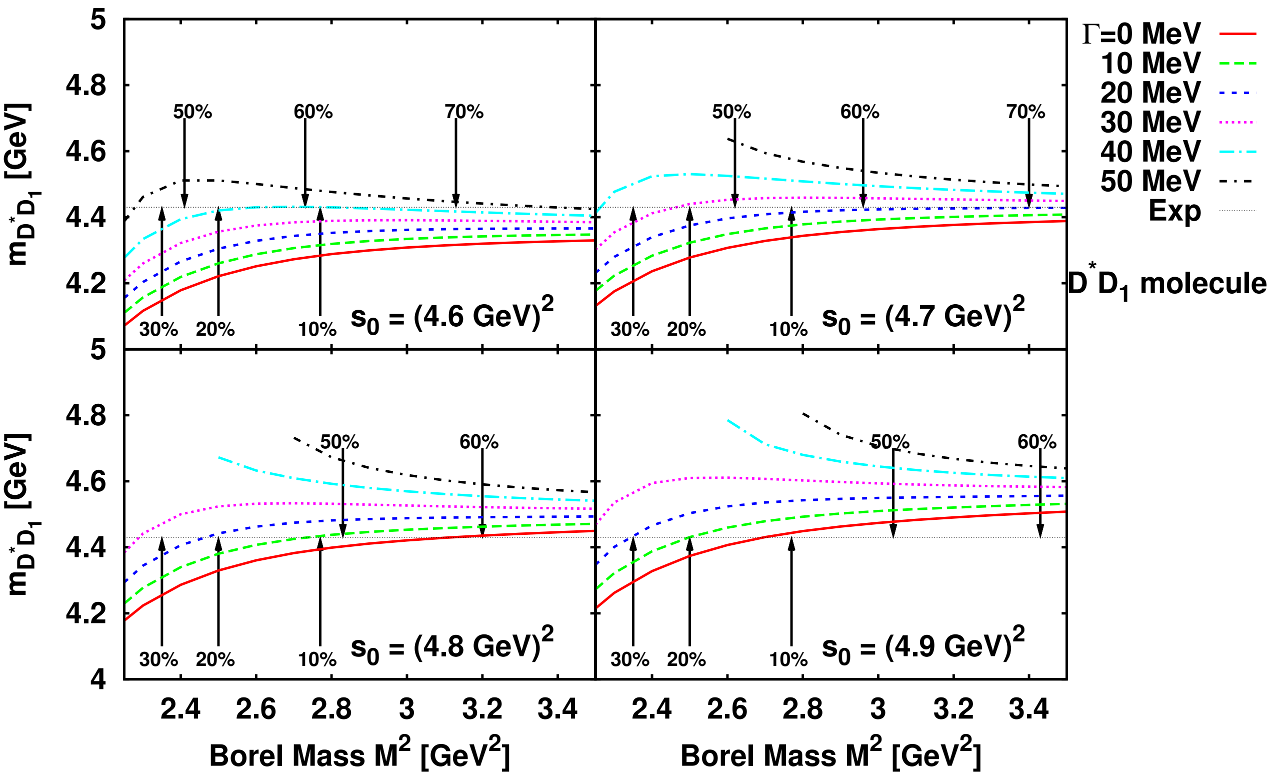

FIG. 5: (color online). Results for D^*D_1 molecule state. Each panel shows different continuum threshold case. Upward and downward arrows indicate the region of the Borel window $M^2_{\rm min}$ and $M^2_{\rm max}$, respectively. Associated numbers in percent denote the dimension eight condensate contribution for upward arrows and continuum contribution for downward ones.

Indeed, M^2 dependence of the mass looks promising because it is a monotonically increasing function of M^2 as in the case of D^*D_1 molecule.

Figure 6 shows the results for the D_1D molecule. As in Fig. 5, we plot the mass for various widths in each panel. The continuum threshold $\sqrt{s_0} = 4.6$ GeV corresponds to the center value in the previous analysis [9]. One can see that the stability is achieved in all the cases. In the largest $\sqrt{s_0}$ case, the optimized width is around 40 MeV, which is a little smaller than the experimental value. Reducing the continuum threshold leads to larger width; for $\sqrt{s_0} = 4.5$ GeV, we obtain the $\Gamma = 60$ MeV. Up to this value, no relaxation of the Borel window criterion is needed. Taking $\sqrt{s_0} \leq 4.4$ GeV makes the width much closer to the experiment, however, we need to relax the condition for the dimension eight condensates and/or the continuum contribution as in the case of $Z(4430)$ to validate QCDSR. In such cases, we can obtain $\Gamma \simeq 80$ MeV ($\sqrt{s_0} = 4.4$ GeV) and $\Gamma \simeq 100$ MeV ($\sqrt{s_0} = 4.3$ GeV). Hence, our analyses support an existence of D_1D molecular state with large width and its manifestation as $Z_2^+(4250)$ resonance.

V. SUMMARY

In summary, we have extended previous QCDSR analyses of exotics to include the total width, by employing the Breit-Wigner function to the pole term. As a general feature, for the cases where the predicted mass for $\Gamma = 0$ increases with increasing Borel mass M^2, introducing the width increases the predicted mass at small Borel mass region, and improves the Borel stability, From this point of view, none of the sum rules based on interpolating currents with tetraquark components are favored. On the other hand, sum rules based on interpolating currents with molecular description as D^*D, D^*D_1 and D_1D, are shown to give valid sum rules for $X(3872)$, $Z^+(4430)$ and $Z_2^+(4250)$ respectively. For $X(3872)$, the inclusion of the width slightly modify the mass, leading to a better agreement with the experimental result. For $Z^+(4430)$ and $Z_2^+(4250)$, molecular description proposed in Refs. [7, 9] are largely improved by introducing the width. We have obtained stable results with $\Gamma_{Z(4430)} \simeq 40$ MeV and $\Gamma_{Z_2(4250)} \simeq 40-100$ MeV. These results strongly support the previous results based on $\Gamma \to 0$ limit, that the $Z^+(4430)$ and $Z_2^+(4250)$ resonances are strong candidates for molecular states. Moreover, we have established that the QCD sum rule with four OPE terms has sufficiently rich structure so that an estimate of the total width is also possible.

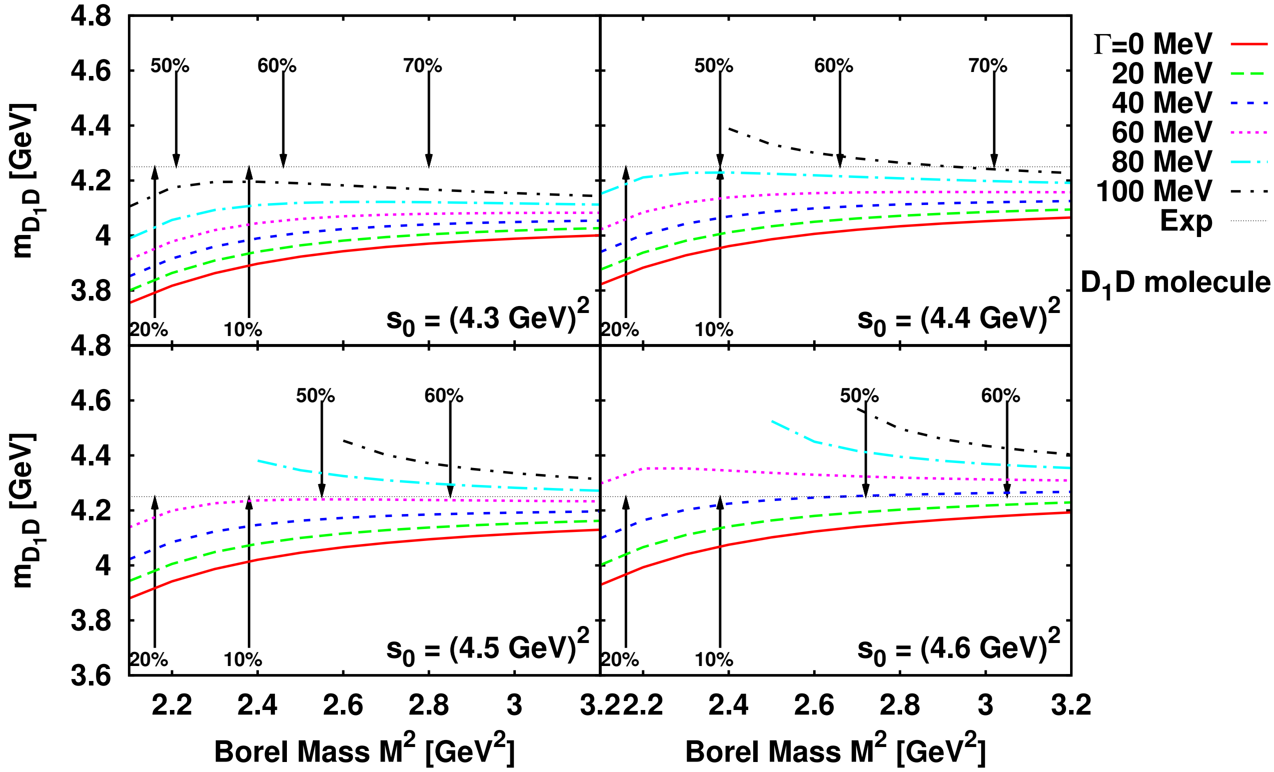

FIG. 6: (color online). Results for D_1D molecule. Symbols are similar to Fig. 5.

Acknowledgment

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